X Marks the Spot:
Unlocking the Treasure of Spatial-X Models

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Abstract

In recent years, political scientists have made extensive use of spatial econometric models to test a wide range of theories. In a review of spatial papers, we find that a majority of these studies use the spatial autoregressive (SAR) model. Although this is a powerful method that reveals inferences about diffusion processes, it is also highly restrictive and makes assumptions that often are not appropriate given the expressed theories. We contend that spatial-X (SLX) models are a better reflection of typical theories about spatial processes. Our simulations demonstrate that SLX models consistently retrieve the direct and indirect effects of covariates when the true data-generating process reflects other spatial processes. SAR models, on the other hand, tend to find phantom higher-order effects that are not present in the data. We further demonstrate how SLX models reveal heterogeneity in patterns of spatial dependence in countries' defense burdens that SAR models cannot discover.

Political scientists have increasingly made use of spatial econometric models. The appeal of these models is that they relax rigid assumptions about the independence of observations across space. While the move toward these techniques is encouraging for the building of more realistic models of politics, there is frequently a substantial disjuncture between theoretical propositions, what is actually being tested, and how results are interpreted. This has particularly been the case when researchers rely only on the spatial autoregressive (henceforth SAR) model to test their theories.

The SAR model captures contemporaneous interdependence in outcomes—how the value of the dependent variable in one unit, y_i, affects the value of the dependent variable in another unit, y_j. Although the SAR model is the most popular spatial model among political scientists, it is a highly restrictive model that researchers should use with caution. We argue that, depending on the theory being tested and the pattern of spatial effects, the spatial-X (henceforth SLX) model may be preferable to the SAR. This is the case because the SLX model allows the spatial processes to influence the outcome through one or more independent variables. Examples of this include trash in a neighbor's yard affecting the value of your home and lax gun regulations in nearby states producing negative externalities (Knight 2013). Since there is no implicit endogeneity, both the estimation and the interpretation of SLX models are considerably easier than with SAR models.

Ideally, selecting the appropriate spatial econometric model should be driven by one's theory about the presence of spatial dependence in the outcomes, observables, and/or unobservables (Cook, Hays, and Franzese 2015). After selecting a theoretically-grounded model, appropriate specification tests (such as a Lagrange multiplier test) should further buttress those choices (Darmofal 2015). In the absence of an appropriately specific theory, one at-first obvious approach would be to borrow a strategy from time series analyses (Hendry 1995): start with a general model that includes all three types of spatial dependence, and then gradually pare down the model by testing restrictions (Vega and Elhorst 2015). A full interpretation of the various total, indirect and direct effects and specification tests would

then complete the process (LeSage and Pace 2009; Whitten, Williams, and Wimpy n.d.). In practice, however, the estimation and interpretation of a general model that includes spatial relationships from all three sources is very difficult and thus it is rarely done.

So what do political science researchers do? In our review of the literature, we found that the overwhelming majority of spatial publications in the discipline start and end with the SAR model. This is the case despite the fact that the underlying theories, as expressed by the authors, often do not match the assumptions that the SAR model implicitly imposes. We also find that researchers rarely interpret the variety of quantities of interest available in the SAR models that they estimate.

In this paper we make a case for an approach to spatial model building that takes advantage of the flexibility and relative simplicity of the SLX model. In this approach, the theoretical implications of different model specifications are quite transparent and the assumptions imposed by the SAR model can easily be turned into testable propositions. This approach simplifies the preliminary stages of model specification, and minimizes inferential errors while maximizing the ease of interpretation. We offer these recommendations knowing full well that they are no substitute for careful theorizing and modeling of the data-generating process (DGP). At the same time, we think that this approach offers a better alternative to the current one that dominates the use of spatial econometrics in political science. By doing so, we hope to illuminate a path forward and provide a practical midway point between current unsatisfactory practice and the ideal modeling approaches advocated by Cook, Hays, and Franzese (2015) and Vega and Elhorst (2015).

In the sections that follow, we begin with a brief overview of SAR and SLX models and point out how they differ in expectations about endogeneity, feedback, and higher-order effects. We demonstrate that SLX is more flexible in terms of producing an empirical test that closely matches underlying theory. We then identify a troubling pattern from our survey of the use of spatial econometric models: theories predicting spillovers among only first-order

neighbors are often tested by SAR models that impose higher-order (and feedback) effects. In a series of Monte Carlo experiments we explore which type of model is more robust to errors in expectations about the DGP; more specifically, what happens when the true model is an SLX, but we estimate an SAR and vice versa. These experiments show that SAR models perform poorly in terms of identifying the correct direction of spatial dependence, and this problem worsens as the degree of spatial heterogeneity increases. SLX models, however, do an adequate job of characterizing spatial effects of processes typically encountered in political science. We then provide an illustration of countries' defense burdens which shows how SLX models can reveal interesting patterns of spatial heterogeneity—both in terms of *how* the countries are connected and *to what extent*—that SAR models cannot. Most notably, we show that instability (in the form of interstate war) spills over into neighbors' defense burdens, and that these effects are above and beyond what one might attribute to positive spatial dependence from a SAR model. We conclude with a discussion of the implications of our findings for future research along with some potential paths forward for model selection.

Modeling Spatial Dependence in Politics

It has been a little more than a decade since political scientists wrote the first papers about the potential for spatial econometric models of political phenomena (Beck, Gleditsch, and Beardsley 2006; Franzese and Hays 2007; Ward and Gleditsch 2008). In the wake of these path-breaking works, there has been a rapid increase in the employment of spatial econometric models by political scientists. Scholars have made theoretical arguments about how spatial relationships help to determine policies (Gray 1973; Simmons and Elkins 2004; Neumayer, Plümper, and Epifanio 2014), conflict (Buhaug and Gleditsch 2008; Garcia and Wimpy 2016), party competition (Williams and Whitten 2015; Bohmelt et al. 2016), terrorism (Midlarsky, Crenshaw, and Yoshida 1980; Neumayer and Plümper 2010*a*) and many other outcomes.

This section provides an overview of two of the most popular spatial econometric models: SAR and SLX models. Spatial dependence occurs in both models, whether it is in the outcomes (SAR) or in the observables (SLX). The researcher specifies the manner in which all the observations are connected to each other via an $N \times N$ weights matrix ($\mathbf{W}$) in both models, so it is worthwhile spending some time on this critical task. For ease of presentation, we explore the basic mechanics of each model in the context of a simple contiguity or first-order weights matrix.[1] In Figure 1 we show a toy example of how these types of matrices are constructed. On the far left side of this figure, we show a map of six units with arrows depicting first-order neighbors. For now, only consider the first-order contiguity weights matrix, $\mathbf{W}$, where each cell identifying a pair of neighboring units contains a one and all other cells contain zeros. Since being a neighbor is symmetrical, this matrix is symmetrical. And since a unit cannot be a neighbor of itself, the main diagonal contains all zeros. As we will see below, these features of $\mathbf{W}$ have meaningful consequences for substantive inferences.

[Figure 1 about here.]

The Spatial Autoregressive (SAR) Model

The equation for a basic SAR model is

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \rho\mathbf{W}\mathbf{y} + \boldsymbol{\epsilon} \tag{1}$$

and the reduced equation (isolating $\mathbf{y}$ on the left side) is

$$\mathbf{y} = (\mathbf{I}_n - \rho\mathbf{W})^{-1}\mathbf{X}\boldsymbol{\beta} + (\mathbf{I}_n - \rho\mathbf{W})^{-1}\boldsymbol{\epsilon}. \tag{2}$$

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[1] In addition to making the presentation more clear, this is the most popular type of weights matrix used in political science research. In our survey of the literature (described below), we found that 72.3% of the models reported used a weights matrix specified based on geography. Regardless, the same types of patterns and problems discussed in this paper persist in SAR models regardless of the specification of $\mathbf{W}$.

From the infinite series expansion of the spatial multiplier,

$$(\mathbf{I}_n - \rho\mathbf{W})^{-1} = (\mathbf{I}_n + \rho\mathbf{W} + \rho^2\mathbf{W}^2 + \rho^3\mathbf{W}^3 + \ldots) \tag{3}$$

we can see that higher order and feedback effects, or *global effects*, are present in any and all SAR models. Consider the second weights matrix depicted in Figure 1 ($\mathbf{W}^2$), which has two meaningful features. First, unlike $\mathbf{W}$, the main diagonal elements—which convey the relationship of a unit with itself—do not equal zero. These values reflect feedback effects where the effect of a unit on its neighbors comes back to affect the unit itself.

Second, we can see that $\mathbf{W^2}$ contains non-zero values for all cells containing second-order neighbors.[2] The result is that the effects extend beyond first- and second-order neighbors (because of $\mathbf{W}$ raised to increasingly higher values in Equation 3), and they occur simultaneously at time t. We can thus label them as "global effects."

While interpreting the coefficients is a reasonable place to start, it is a bad place to stop in the interpretation of an SAR model. Given all of the different higher order and feedback terms implicit in $\mathbf{W}^2$ and the higher order terms in Equation 3, a general interpretation of the estimated ρ masks substantial variation in the estimated effects across units. For instance, if we want to infer the effect of a single x on y, then one approach is to examine the partial derivatives matrix (LeSage and Pace 2009; Whitten, Williams, and Wimpy n.d.):

$$\left[\frac{\partial \mathrm{E}(y)}{\partial x_1} \cdots \frac{\partial \mathrm{E}(y)}{\partial x_N}\right] = (\mathbf{I} - \hat{\rho}\mathbf{W})^{-1}\hat{\beta} \tag{4}$$

where the resulting $N \times N$ matrix (N is the total number of observations) contains both the impacts of x_i on y_i, or direct effects (along the diagonal), and the impacts of x_i on y_j, or indirect effects (along the off-diagonal).

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[2]The value in these cells reflects the number of paths through which each pair of units are second-order neighbors. So, for example, $\text{cell}_{AE} = \text{cell}_{EA} = 1$ because A and E are only connected through B, but $\text{cell}_{BC} = \text{cell}_{CB} = 2$ because B and C are connected through both A and D.

The Spatial-X (SLX) Model

In contrast to the SAR model, the SLX model offers a framework in which researchers may choose whether to model local or global spatial relationships. In addition, the SLX model is relatively easier to estimate and interpret. In practice, the SLX model has been most often discussed as a modeling strategy for exogenous spatial effects from a direct neighbor. This effort has primarily been driven by the work of LeSage and Pace (2009) in which they suggest that the SLX model works best in the case of "externalities," or "local spillovers". Spillovers would appear to accurately characterize many political science phenomena; something happens to a neighbor that affects the outcome of interest.

The equation for a basic SLX model can we written as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\mathbf{Z}\boldsymbol{\theta} + \boldsymbol{\epsilon} \tag{5}$$

where $\mathbf{Z}$ is the matrix of variables expected to exert spatial influence on $\mathbf{y}$ through a theoretically specified $\mathbf{W}$ matrix that connects observations to each other through a vector of spatial parameters $\boldsymbol{\theta}$.[3]

If we start with simple SLX model with a single independent variable, x_1, such that $\mathbf{X} = \mathbf{Z} = x_1$, we calculate the effect of x_1 on y as

$$\left[\frac{\partial \mathrm{E}(y)}{\partial x_1} \right] = \hat{\beta}_1 + \hat{\theta}_1 \mathbf{W}. \tag{6}$$

In contrast to the infinite series expansion in the SAR spatial multiplier, the indirect effect $(\hat{\theta}_1 \mathbf{W})$—or "neighbor effect"—is only present at the first-order. In other words, with the specification presented in Equation 5, the effect of x_1 is limited to only local effects; the effect does not continue to second-order neighbors, and there are no feedback effects. However, as

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[3]We use $\mathbf{Z}$ to reflect the possibility that $\mathbf{Z}$ and $\mathbf{X}$ can have different contents.

we discuss below, if we want to specify higher order and feedback effects, we can incorporate them into an SLX model.

Without realizing it, scholars often use SLX models by including spatially-weighted independent variables. In fact, any model that controls for the sum or average of neighbors' attributes reflects an SLX specification (Drolc, Gandrud, and Williams 2019). The most common example incorporates a temporally-lagged spatial lag (TLSL), which tests a common argument in theories of policy diffusion that the diffusion process—or the influence of neighbors' covariates more generally—occurs with a temporal lag. This variant of the SLX equation can be written as:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\mathbf{y}_{t-1}\theta + \boldsymbol{\epsilon} \tag{7}$$

While the estimation is straight-forward, the presence of the TLSL adds a temporal dimension to the quantities of interest.[4]

In the next section, we provide some guidelines as to when to use one model over the other by highlighting each model's flexibility and ability to derive correct causal inferences.

Theory, Specification, and Estimation in SAR and SLX Models

The general motivating force behind spatial econometric models is to test theories about how y_i is a function of some aspect of unit j. If a researcher's expectations are that the

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[4]A common misconception is that this is "an alternative specification of the spatial autoregressive model" (Beck, Gleditsch, and Beardsley 2006, 40). However, the TLSL forces the causality to go in a single direction (i.e., from y_{t-1} to y_t), which eliminates the defining feature of SAR models: spatial endogeneity. Neumayer and Plümper (2010b, 158) note that a model with a temporal lag "is not strictly speaking a spatial lag model". Feedback for i from x_{t-1} can occur by first influencing i's neighbors at time t, which then influences j's neighbors (including i) at time $t+1$. In the first section of our Supplemental Materials document, we provide some more discussion of the features of the TLSL model and why it is more like an SLX model than a SAR model.

spatial relationships are between y_i and y_j, and that these relationships are global (including feedback from y_i to y_j and back to y_i) and occur immediately, then the SAR model is clearly the appropriate model to choose. On the other hand, if a researcher's expectations are that the spatial relationships in their study are between y_i and some variable z_j, and these relationships are local in nature, then the SLX model is clearly the appropriate model to choose. But what about a researcher whose expectations are less sharp? Under these circumstances, we echo the suggestion by Vega and Elhorst (2015) that the SLX model is a better place to start than the SAR. The advantages of the SLX model are four-fold: greater flexibility in specifying lower-order (local) versus higher-order (global) effects, the ability to relax and test common factor restrictions, the ease of estimation and interpretation, and more flexible and realistic specifications of temporal processes. We will explore each of these in turn.

First, the SLX model offers a more flexible approach to modeling lower-order versus higher-order spatial effects than the SAR model. As we outlined above, the SAR model implicitly imposes a number of strong assumptions that may not be immediately obvious to scholars, and that often run contrary to their theoretical expectations about the spatial processes at work. In most incarnations of the SAR model, there is no way to limit spatial effects to only local first-order effects.[5] Although the SAR model certainly captures *local* effects, by construction it also finds higher-order effects. The infinite series expansion (Equation 3) reveals that all observations are eventually impacted by any change in x; this is the case even if there is no theoretical reason to expect that 9th-order neighbors would be influenced (albeit in a small fashion) by a change.[6] The SAR model offers no flexibility in terms of how many orders of neighbors are impacted. Feedback effects start in the second order (e.g., $\mathbf{W^2}$ in Figure 1). Though one may theorize that the total effect of a change in x_j for observation i arises only from first-order local effects, the SAR model forces there to

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[5]In the case of SAR models using directed dyadic data, one can limit the effects to the first-order by distinguishing between the *sources* and *targets* of stimulus (e.g., Neumayer and Plümper (2010*b*, 152–154)).

[6]The one exception to this would be for a unit that has no connectivity with any other. In Figure 1 such a unit would appear as an island with values equal to zero for the relevant column and row in $\mathbf{W}$.

be higher-order and feedback effects, whether this accurately reflects the true spatial process or not. The consequence is that SAR models will risk finding a *global* process even if one is not present.

In an SLX model, expectations about local first-order processes can be easily tested through the specification of a model like that in Equation 5. But can SLX, on the other hand, perform adequately when the true spatial process includes higher-order effects? The answer is "yes." By altering the model specification, scholars can incorporate higher-order effects into their models of spatial processes. For example, imagine that we have an expectation of higher-order effects that stop at the 3rd-order. For ease of exposition, we limit our model to a single independent variable $\mathbf{x}$. In this case, we would estimate an SLX model specified as

$$\mathbf{y} = \beta\mathbf{x} + \theta_1\mathbf{W}\mathbf{x} + \theta_2\mathbf{W^2}\mathbf{x} + \theta_3\mathbf{W^3}\mathbf{x} + \boldsymbol{\epsilon}.^{7} \tag{8}$$

The total estimated effect of x on y would be

$$\left[\frac{\partial \mathrm{E}(y)}{\partial x_1} \cdots \frac{\partial \mathrm{E}(y)}{\partial x_N}\right] = \hat{\beta} + \hat{\theta}_1\mathbf{W} + \hat{\theta}_2\mathbf{W}^2 + \hat{\theta}_3\mathbf{W}^3 \tag{9}$$

where $\hat{\beta}$ is the estimated direct (or zero-order) effect of x, $\hat{\theta}_1$ is the estimated first-order indirect effect, $\hat{\theta}_2$ is the estimated second-order indirect effect, and $\hat{\theta}_3$ is the estimated third-order indirect effect. It is worth noting that restrictions on the order of effects can be tested, so one strategy is to start with an n-order specification and then use hypothesis tests to pare down the model.[8]

With the model specification shown in Equation 8, the SLX model mimics the SAR model in its estimation of feedback effects. However, there might be situations where one expects higher-order neighbor effects but does not expect that the spatial process will have feedback

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[7]In this specification, $\mathbf{X} = \mathbf{x}$, a single vector containing the values for independent variable x and $\mathbf{Z} = (\mathbf{x} \quad \mathbf{x} \quad \mathbf{x})$.

[8]One word of caution is that if there are few neighboring observations in the weights matrix, then higher-order representations of $\mathbf{W}$ can exacerbate problems of multicollinearity and inflate standard errors.

effects. In these cases, one can exchange the terms with $\mathbf{W}$ squared, cubed, and raised to higher powers for n-order contiguity matrices such as $\mathbf{W}_{2nd}$ in Figure 1 (as we demonstrate in the *Supplemental Materials* document). One can also test whether the spatial process exhibits feedback loops by comparing the model fit (through information criterion such as AIC and BIC) of an SLX specification with traditional squared and higher order $\mathbf{W}$ terms versus that from one with nth-order contiguity matrices.

The second relative advantage of the SLX model is the ability to relax and test common factor restrictions. In the case of an SAR model, there is an imposed assumption that all of global spatial dependence works through a single spatial parameter, ρ. For example, consider estimating an SAR model where we have two independent variables, x_1 and x_2 in $\mathbf{X}$. Since there is only one estimated ρ in the model, the infinite series expansion of the spatial multiplier (found in Equation 3) is identical and the only difference in the estimated spatial effects comes from different coefficient estimates in $\hat{\boldsymbol{\beta}}$. In other words, the strength of spatial dependence, and the manner in which higher-order effects reverberate throughout the system, is identical for both x_1 and x_2 by construction. Moreover, one spatial autocorrelation coefficient (ρ) is used to represent the declining spatial effects across all orders of neighbors, albeit at an order of magnitude smaller at each additional order of neighbors (i.e., ρ, ρ^2, ρ^3, and so on). Rather than this common factor restriction being imposed by the model (in the case of SAR), in the context of an SLX model it becomes a testable proposition; we can test the accuracy of these restrictions and modify the specification accordingly.[9]

It is not outside the realm of possibility to consider political science applications in which spatial relationships may vary across independent variables. In such cases of spatial heterogeneity, the SLX model offers a far more straight-forward and accurate approach by allowing scholars to estimate different spatial parameters for the independent variables. Unlike the SAR model, the SLX model also provides the flexibility of spatially lagging only

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[9]For example, in Moore and Shellman (2007)'s analysis on refugees' destination, they estimate different parameters for the spatial contiguity effects of a number of independent variables.

those variables from a neighboring unit that are theorized to impact y_i via some form of spatial dependence. In other words, **X** and **Z** do not have to be the same. Some covariates might only have direct effects (i.e., only be in **X**), and others might only have indirect effects (i.e., only be in **Z**). Furthermore, the **W**s can vary across the different z variables (if different interconnectivities are theorized to exist), and this model allows for spatial heterogeneity (e.g., when $\theta_1 \neq \theta_2$). As opposed to the complicated multiple SAR model, estimation for an SLX model with different interconnectivities is rather straightforward.

The third advantage of SLX models to applied researchers is ease of estimation and interpretation. Perhaps the most notable argument for the SLX model comes from the Gibbons and Overman (2012) critique of the general enterprise of spatial econometrics. These authors argue that analysts using the SAR model end up with models that are weakly-identified due to the hurdles involved in estimation of these endogenous models. The authors further point out that the model being estimated rarely fits the expressed theory. As such, they make an argument for the far-simpler, exogenous SLX model.

SLX models provide much more meaningful coefficients in terms of allowing scholars to quickly and correctly make inferences about the spatial processes at work. While the SAR model produces a *global* coefficient of average spatial dependence (ρ), getting substantive impacts and estimates of uncertainty from other coefficients of interest is notably difficult. Moreover, the ρ does not easily distinguish between local and higher-order connectivity, thus making it hard to make substantive inferences about the degree of spatial dependence from one neighbor to another. Given the complexity of calculating quantities of interest from SAR models, it is perhaps understandable that few scholars move beyond simply interpretations of the sign and significance of ρ.

In contrast, the SLX model is easier to interpret. The biggest difference in interpretation is that in an SLX model, the respective $\hat{\beta}$ can be interpreted as is; $\hat{\beta}$ is the estimated change in y_i for a one-unit increase in x_i. On the other hand, in an SAR model, the $\hat{\beta}$ represents

the estimated "direct effect" of x_i on y_i, which, due to spatial dependence in the outcomes, we never actually observe because they are merely the pre-spatial effects that have yet to be filtered through the spatial dependence. This is complicated by the fact that the effect of x_i on y_i also depends on the strength of feedback. The spatial coefficients from an SLX model, the $\hat{\theta}$ terms, are interpreted as the estimated impact of a one-unit increase in the relevant z variable on the outcome variable in a neighboring unit.[10] In comparison, this effect is difficult, if not impossible, to ascertain from merely examining the $\hat{\rho}$ value from an SAR model.

The fourth advantage of SLX models is that they offer more flexible and realistic specifications of temporal processes. Up until now, we have ignored possible temporal variations in our data other than to note that by construction SAR models impose that all effects of y_j on y_i and feedback effects from y_i back on itself happen immediately. If our theory is that the effect of y_j on y_i and any associated feedback effects take time to happen, we can test this through an SLX model by including $\mathbf{y}_{t-1}$ as a term in our specification of $\mathbf{Z}$ (see Equation 7).

In the next section, we explore patterns in how political scientists have selected and interpreted spatial econometric models.

Spatial Dependence in Practice

To provide a systematic overview of how researchers in political science have used spatial econometric models, we reviewed every work published through the end of 2015 that cited Beck, Gleditsch, and Beardsley (2006) and/or Franzese and Hays (2007)—two highly in-

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[10]As we mentioned above, we are limiting the present discussion to cases where $\mathbf{W}$ is a row-standardized contiguity matrix. With more complicated $\mathbf{W}$ where the weights matrix contains values other than 0 and 1, the estimated effect of a one-unit increase in x_j on y_i is $\hat{\theta}$ times w_{ij}.

fluential early papers promoting the use of spatial models.[11] We coded the use of these techniques in the main empirical model that was reported.

[Table 1 about here.]

Table 1 shows a taxonomy of the use of spatial econometric models by political scientists in terms of the type of spatial theory expressed and the model estimated. There are three patterns worth noting in this table. First, although SAR models recover global effects—or those where all non-isolate observations are influenced by changes in one observation—only 8.9% (5 of 94 studies) expressed theories that are global in nature. In those rare cases when scholars explicitly state global theories, on the other hand, they tend to appropriately test those theories with a model based on dependence in the outcomes (i.e., SAR or a more complex model that includes an SAR process). Second, when political science researchers have expressed a theory that is either local or not specific in terms of the types of spatial expectations, almost half the time (46.1%) they have estimated an SAR model which imposes global spatial effects.

Third, only 10.6% (10 out of 94 studies) of the publications we reviewed feature more than one pattern of spatial dependence. There is also generally little discussion of the process—if any—used by scholars to pare down more complex models to simpler models. Altogether, this is indicative of scholars beginning with a specific model in mind (usually the SAR), rather than a general-to-specfic approach. We return to this point in our discussion of paths forward below.

Among those papers in our survey that estimated an SAR model, the overwhelming majority offered a theory of indirect effects—the effect of x_i on y_j—that were local in nature rather than global. In fact, only 8.9% (4 out of 45 studies) of those that estimated an SAR model explicitly expressed a global theory. Essentially this means that scholars theorize that

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[11]This initial search yielded 155 publications. We narrowed this down to 94 studies that reported results from at least one spatial econometric model.

explanatory variables may have an effect on first-order neighbors (local) but are silent about potential impacts on higher-order neighbors (global) (Elhorst 2014). This latter finding is striking because the SAR model imposes global relationships across all spatial units. In contrast, spatial relationships in SLX models can be specified as either local or global.

Another common pattern emerges in our in-depth exploration: scholars often estimate a series of SAR models, each with a different specification of the weights matrix (e.g., Flores 2011; Gassebner, Gaston, and Lamla 2011; Goldsmith 2007; Obinger and Schmitt 2011).[12] Recall that the SAR model imposes a common factor restriction that all of the spatial dependence operates through one parameter, ρ, connected through a properly specified $\mathbf{W}$. An SLX model, on the other hand, easily allows different variables to influence the outcome through multiple spatial dependence paths; the final model can then be pared down based on traditional model selection criteria. The examples referenced above highlight at least three problems that arise from estimating separate SAR models: first, it severely limits whether authors can determine which model best approximates the data-generating process. Second, if multiple weighting schemes are operating simultaneously and are correlated, then any model including only one will be biased. Third, even if only one weights matrix correctly specifies the spatial process, if explanatory variables operate through the weights matrix in different ways, then the SAR model will not be able to disentangle that spatial heterogeneity.[13]

In the next section we present the results from two sets of Monte Carlo experiments in which we explore how SLX and SAR models perform under different data generating processes (DGPs).

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[12]These examples do not represent a comprehensive list. Instead, we identify them as representative examples of instances where the inferences might be different with an SLX model, either due to methodological differences or because the SLX model represents a more appropriate way of testing the theoretical expectations.

[13]This is not to say that there are no good examples of scholars carefully selecting the appropriate empirical model and then painstakingly exploring the substantive meaning of those models; Freeman and Quinn (2012: 67), for example, reveal how income inequality and financial integration influence democracy through multiple channels of spatial dependence.

Experiments

In this section we provide two sets of Monte Carlo experiments to explore the robustness of both models to errors in expectations about the DGP. In the first set of experiments, we expect that SLX models will perform admirably in deriving inferences about the effects of explanatory variables, even though the true process is generated by an SAR. One of the advantages of SLX models that we described above is that they can mimic the higher-order effects of SAR models in situations that scholars are likely to encounter in practice. The opposite, however, is unlikely to be true. In the second set of experiments we expect that SAR models are too inflexible to effectively deal with spatial heterogeneity that often accompanies an SLX DGP.

SAR Data-Generating Process

In the first set of experiments we generate data using the reduced-form equation for the SAR model found in Equation 2, with matrix $\mathbf{X}$ containing a single variable drawn from a uniform distribution, $x \in [-10, 10]$, and where $\beta = 1$. $\mathbf{W}$ is an $N \times N$ symmetric row-standardized contiguity weights matrix, where each element below the diagonal is randomly drawn from a Bernoulli distribution. We simulated 1000 data sets at each of nine different scenarios defined by the strength of the spatial autocorrelation, $\rho \in \{-0.8, 0.8\}$.[14] In each section of Table 2, we display our findings from estimations using a different SLX specification.[15]

The first SLX model that we estimate includes one spatially-lagged independent variable ($\mathbf{Wx}$) and is thus specified as

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[14]It is worth noting, however, that values of ρ with an absolute value close to 1 are exceedingly rare in political science research.

[15]In Table 1 of our appendix we present the same results for an SAR estimation. Not surprisingly, the SAR model does an excellent job of recovering the true DGP.

$$\mathbf{y} = \mathbf{x}\beta + \mathbf{W}\mathbf{x}\theta + \boldsymbol{\epsilon}. \tag{10}$$

The βs are not directly comparable across SAR and SLX specifications; instead, with the use of the partial derivatives matrix one can easily compare the average direct, indirect and total effects across models (LeSage and Pace 2009). Additionally, since these effects can be partitioned into n-order effects, we assess the performance of the model at different orders. Table 2 shows how often the estimated SLX model specification's 95% confidence intervals include the true effects given the DGP characterized in Equation 2. Each section of the table provides the recovery rates for a different SLX model specification (more on the others below), and each column represents a different strength of spatial lag coefficient (ρ in Equation 2).

[Table 2 about here.]

There are two clear patterns in the top section of Table 2. First, the simple SLX model recovers the true average direct effect at an acceptably high rate (nearly 95%) for all values of ρ except 0.8.[16] The recovery rates for the zero-order direct effect (characterized by the β_{SLX}), on the other hand, dip below 95% at ρ values lower than -0.4 and higher than 0.4. This means that β_{SLX} is capturing the true average direct effect at common values of ρ, but it is not an accurate reflection of the zero-order direct effect (β_{SAR}). The second clear pattern is that the inclusion of one $\mathbf{W}x$ term is enough to recover the true first-order indirect effects nearly 95% of the time for all $\rho \in [-0.6, 0.6]$, but overall this model specification does a poor job of capturing the true total indirect effects. These two patterns are a consequence of an overly-simplistic SLX model specification given the true DGP. The lack of higher-order $\mathbf{W}$ matrices means that there cannot be second-, third-, or higher-order indirect effects and that there can be no feedback effects. This is why the simple SLX in the top section of

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[16]It is worth noting that negative values are quite rare and that we have never seen such a high value for ρ in a political science application.

Table 2 has a recovery rate for all higher-order direct and indirect effects that is 0% at all values of ρ (represented by dashes).

These results from the experiments displayed in the top section of Table 2 should certainly warrant caution from scholars. To what extent can we eliminate some of these concerns by modifying the SLX model specification? In the second set of experiments, we add a first-order weights matrix squared ($\mathbf{W}^2$) and estimate a second SLX coefficient (θ_2) with a model specified as

$$\mathbf{y} = \mathbf{x}\beta + \mathbf{W}\mathbf{x}\theta_1 + \mathbf{W^2}\mathbf{x}\theta_2 + \boldsymbol{\epsilon}. \tag{11}$$

Recall that $\mathbf{W}$ matrices raised to higher-orders allow for spatial effects that cycle through neighbors of neighbors, and back to the originator (see the first-order squared matrix in Figure 1). This second model specification, therefore, should help address the poor performance of the first model in estimating the average indirect effects created in the DGP.

From the second section in Table 2, we can see that the addition of the squared weights matrix improves the performance of the model in all of the weak spots detailed above. First, the zero-order direct effects are now recovered at 95% for nearly all the values of ρ, in addition to the second-order direct effects (feedback effects), which results in even more accurate estimates of the average direct effect. Second, the ability to model second-order indirect effects not only allows one to gain accurate inferences about those effects (nearly 95% recovery rates at all values), but also about the average indirect effects overall. Instead of poor performance at values outside of $\rho \in [-0.2, 0.2]$, the average indirect effects are well-recovered at a much broader range of values ($\rho \in [-0.6, 0.4]$). Furthermore, in these experiments the improvement in model performance does not come at a cost of multicollinearity; on average the highest variance inflation factor (VIF) is a little more than 2.[17]

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[17]Of course, scholars must make their own decisions as to whether the increased multicollinearity in their model is worth being able to derive more accurate inferences about spatial processes.

As we discussed earlier, the infinite series expansion of the spatial multiplier in the SAR model (see Equation 3) means that the indirect effects increase forever, but at a declining rate. To mimic this sort of process with an SLX model, one would have to include a large number of higher-order weights matrices. The utility of this approach, however, would be limited as the size of the indirect effects from higher-order effects is quite small, and this would likely exacerbate any issues of multicollinearity. The next step, then, is to see if including a cubed weights matrix effectively recovers the direct and indirect effects by estimating a model specified as

$$\mathbf{y} = \mathbf{x}\beta + \mathbf{W}\mathbf{x}\theta_1 + \mathbf{W^2}\mathbf{x}\theta_2 + \mathbf{W^3}\mathbf{x}\theta_3 + \boldsymbol{\epsilon}. \tag{12}$$

The recovery rates presented in the bottom section of Table 2 reveal the untapped promise of the SLX model even in those processes ripe with spatial diffusion. In all but the most extreme cases of positive spatial autocorrelation (i.e., when $\rho > 0.6$), the SLX model with both squared and cubed $\mathbf{W}$ terms recovers the average direct effect, average indirect effect, and by implication, the average total effect. Thus, in the vast majority of cases that scholars observe in practice, one could estimate an SLX model and make the same substantive inferences regarding the impact of a variable on the observation, its neighbors, and the other observations. The only drawback is higher multicollinearity (the average VIF score is now above 5), which leads to somewhat inflated standard errors. This is certainly not a reason to avoid estimating the SLX model, and is a similar decision to those made by scholars choosing whether to include lower-order terms in interactive models.[18]

In the next section we explore the opposite scenario; where we have an SLX DGP, but we estimate an SAR model.

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[18]In the *Supplemental Materials* document, we also present the performance of an SLX model with a matrix specified as $\mathbf{W_{2nd}}$ in addition to $\mathbf{W}$.

SLX Data-Generating Process

Our next set of experiments assesses the performance of SAR and SLX models when the spatial processes operate through dependence in the observables and there are no higher-order spatial effects. In other words, how do the models perform when the data are generated in a manner consistent with a SLX model? As before, we choose to compare the performance of SLX and SAR models based on whether they can recover the true average direct and indirect effects, calculated via the partial derivatives approach.

We expect that SLX models—not surprisingly—will perform well.[19] Our expectations for SAR models, on the other hand, are mixed. We expect the SAR to do poorly in terms of recovering the true indirect effects because it will find evidence of higher-order and feedback effects in the data that we know do not exist. We also expect that situations of spatial heterogeneity—when the explanatory variables operate through different spatial processes—will be particularly problematic for recovering the correct estimates of direct and indirect effects. We generate data for our first set of SLX DGP Monte Carlo experiments as

$$\mathbf{y} = \mathbf{x_1}\beta_1 + \mathbf{x_2}\beta_2 + \mathbf{W}\mathbf{x_1}\theta_1 + \mathbf{W}\mathbf{x_2}\theta_2 + \boldsymbol{\epsilon}, \tag{13}$$

where x_1 and x_2 are drawn from uniform distributions, $x_1 \in [-10, 10]$ and $x_2 \in [-5, 5]$, and β_1 and β_2 are set to 1. $\mathbf{W}$ is an $N \times N$ symmetric row standardized contiguity weights matrix, where each element below the main diagonal is randomly drawn from a Bernoulli distribution. In order to assess how these models perform under varying degrees of spatial heterogeneity, we conduct 1000 simulations of nine scenarios where we vary the magnitude of the spatial coefficients (θ_1 and θ_2).

The first step is to determine whether the SAR can recover the true total effects of x_1 on $\mathbf{y}$. Figure 2 shows histograms for 1000 estimated average total effects (with 95% confidence

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[19]Results from these estimates are shown in Table 2 of our *Supplemental Materials* document.

intervals via the percentile method depicted with vertical dashed lines), as we vary the values of θ_1 and θ_2. More specifically, the histograms are arranged to represent variation on two critical elements of these models. Moving vertically depicts varying levels of *spatial effects*, ranging from weak (bottom row) to moderate (middle row) to strong (top row). Moving horizontally depicts varying levels of *spatial heterogeneity*, ranging from none (left column) to moderate (middle column) and substantial (right column). In each experiment, the true total effect is the sum of the direct effect ($\beta_1 = 1$) and the indirect effect (θ_1) for x_1, or 0.8, 1.4 and 0.2 (going from the bottom row to the top row). If the 95% confidence intervals (dashed vertical lines) for the calculated total effects from the SAR include the true total effects from the DGP (solid vertical lines), then we can conclude that estimating the SAR model when the true DGP is an SLX will recover the substantive effects of x_1.

[Figure 2 about here.]

Figure 2 shows that both spatial effect size and spatial heterogeneity have meaningful influences on the degree of bias for the SAR under these SLX DGPs. In nearly all of the scenarios, the true total effect appears far away from the bulk of the estimated total effects from the simulations. As one goes from no spatial heterogeneity (first column) to moderate and substantial heterogeneity (third column), the true total effect is less likely to be located near the center of the 95% confidence interval. Thus, when there are multiple patterns of spatial dependence in one model, estimating an SAR model (with one **W**) is likely to provide incorrect inferences about x_1.

Moving up the rows (from weak to strong spatial effect size), the consequences for inferences become even more severe; while the SAR confidence intervals include the true total effect under weak and moderate (with the true total effect falling near the edges of the intervals for substantial heterogeneity) spatial effects, the true total effect is much smaller than those predicted by the SAR model under strong spatial effects. This should cause scholars

to be cautious about using SAR models when either they expect strong spatial dependence or when they expect multiple spatial processes are at work.

While Figure 2 provides clear evidence of overestimation of the total effects, it does not identify whether the source of overestimation is related to the direct effects or indirect effects. Our expectation was that it would be caused by the latter, since the estimation of the direct effect is rather straight-forward in both types of models. Indeed, across all scenarios, the SAR model is able to recover the true average direct effects in at least 95% of the simulations.

Another source of bias in the SAR's calculation of indirect effects is the common factor restriction that forces all of the independent variables to influence the dependent variable via the same spatial multiplier, whether this makes sense or not. We can imagine circumstances where there are different causal processes (or at least different functional forms) operating across the explanatory variables. Figure 3 confirms our suspicions about the dangers of the common factor restriction in circumstances of substantial heterogeneity. The 95% confidence interval only includes the true indirect effect for x_2 (in this case, θ_2) in five of the nine scenarios. More troubling is the fact that under substantial spatial heterogeneity (last column), scholars would conclude that the indirect effects of x_2 are negative (positive), when in fact the true spatial pattern is positive (negative) spatial dependence. Because there are both positive and negative spatial patterns present in the DGP that the SAR model cannot disentangle, there is a risk that scholars will falsely conclude that a spatial pattern exists that is completely opposite of the true pattern.

[Figure 3 about here.]

We suspect that the inability of the SAR model to accurately predict indirect effects in some cases is due to the "phantom" higher-order and feedback effects found by the SAR model. Recall that our experiments are structured so that there are no higher-order effects; by only including one θ for each variable, the indirect effects are limited to being first-order effects (i.e., from first-order neighbors of i). As a result of the SAR model's infinite expansion

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of the spatial multiplier and the fact that weights matrices are treated as polynomials rather than higher-order contiguity matrices, the total effects of x on y will be distributed across orders of neighbors. Thus, the SAR model may find higher-order effects, whether they are there or not.

Figure 4 shows the average estimated second- (top row) and third-order (bottom row) indirect effects for x_1 from the SAR model. The three columns reflect three scenarios of weak, moderate and strong spatial effects (under no spatial heterogeneity), and the solid vertical line shows the true higher-order effects (0). The histograms in Figure 4 demonstrate that the SAR model will often incorrectly discover higher-order (and feedback) effects when they are not actually present (with the lone exception being the third-order effects under weak spatial effects). Indeed, the size of these phantom effects increases with the strength of spatial dependence. The issue is two-fold. First, the DGP is strictly exogenous, so there is no reason for unit i's neighbors to simultaneously influence unit i. Second, there are no higher-order effects within the DGP. The SAR model thus produces inferences about feedback and global effects that are not warranted by the true DGP. It is clear that there is a substantial inferential penalty for estimating an SAR model on an SLX DGP.

[Figure 4 about here.]

Application

If scholars believe that strategic responses occupy a central place in almost any explanation of military expenditures (e.g., Richardson 1960; Palmer 1990; Powell 1999), then spatial econometric models are appropriate. The advancement of spatial econometric models means that scholars can model the precise manners in which choices (such as military expenditures) or conditions (such as instability) spill over into neighboring countries (see, for instance, Shin and Ward 1999; Flores 2011; Plümper and Neumayer 2015). Of course, theory might suggest

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that defense burdens in i directly influence defense burdens in i's neighbors through a single global autocorrelation coefficient (SAR); or, defense burdens in i influence defense burdens in i's neighbors through multiple patterns of connectivities and/or with a temporal delay (SLX); or, defense burdens in i are unrelated to defense burdens in i's neighbors (non-spatial OLS). Though theory can guide this decision, the true process driving military spending is unknown. We argue that a reasonable starting place given this inherent uncertainty is the SLX model since it provides a more flexible specification for estimating spatial heterogeneity and because it avoids some of the problematic assumptions hidden in SAR models.

To provide an applied example of the modeling choices that we discussed and simulated above, we collected data from 1953–2008 in 193 countries, both developed and developing, democratic and authoritarian. We measured defense spending in a manner similar to other scholars (Whitten and Williams 2011; Phillips 2015) as a country's defense burden, or military expenditures as a percentage of GDP. We use the measure developed by Phillips (2015), who uses expenditure data from both the Stockholm International Peace Research Institute and the Correlates of War national material capabilities data (Singer and Small 1972) divided by GDP data from the Penn World Table Version 6.3 (Heston, Summers, and Aten 2009). Both theories of budgeting and previous research tell us that defense burdens are highly autoregressive; the budget in one year is highly dependent on the previous year, plus responses to short-term fluctuations.[20] As a result, the defense burden variable is non-stationary in most countries.[21] This poses a substantial problem for inferences since it increases the risk of spurious results (Granger and Newbold 1974). Since defense burdens are first-order integrated, taking the first difference makes the series stationary. To further

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[20]This is a general feature of budgeting processes in organizations. As Ostrom and Marra (1986, 822) note, "given the complexity of the behavior under examination and the cognitive limitations of decision makers, it is highly unlikely that the budget decisions of any of the organizations are rebuilt from zero each year".

[21]We estimate separate augmented Dickey-Fuller tests for each country with a long enough time series to get stable test statistics. In the original *defense burden* variable we can reject the null hypothesis of a unit root at the 90% confidence level in only 29% of the countries. We reject the null hypothesis in 99% of the countries for the Δ*defense burden*$_{t-1}$ series.

control for the dynamics of defense burden, we include the level in the previous year (*defense burden$_{t-1}$*) and the previous change (Δ*defense burden$_{t-1}$*).[22]

For the purposes of comparing different ways of estimating spatial econometric models, we begin with a simple model of defense burden. We include *total population (logged)* to account for larger countries having larger defense burdens. We also take into account the impact of domestic and international instability by including a dichotomous variable representing the presence of a civil war that is an armed conflict resulting in at least 25 battle-related deaths in a year (Allansson, Melander, and Themner 2017), and interstate war (coded 1 if the hostility level of the militarized interstate dispute reaches 5). We expect that both interstate wars and civil wars will have a meaningful impact on defense burdens. A large portion of the time period under examination occurs in the Cold War, where military spending is driven by alliance commitments and actions of the two superpowers (for a review of this literature, see Goldsmith (2003, 557–559)). We thus believe that the degree of responsiveness to the superpowers' changes in spending will be conditioned by the presence of an alliance with the US (Plümper and Neumayer 2015). To take this into account, we include two interaction variables (and their lower-order terms) made up of *alliance with US* (coded 1 if the state has an alliance with the US according to the Correlates of War), Δ*US military expenditures*, and Δ*USSR/Russian military expenditures.* Finally, Plümper and Neumayer (2010) demonstrate that unmodeled spatial and temporal shocks can falsely suggest spatial dependence. To guard against this possibility, we include *trend* (which is coded 1 for 1950 and increases each year), a dichotomous variable for *1992*[23], and regional variables. Though there are a variety of ways of specifying the connections between countries (for some alternatives, see below), for

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[22]There is a considerable debate about the casual use of lagged dependent variables to control for temporal dependence (see Wilkins (2018) for an overview of this debate).

[23]In a model with annual fixed effects, 1992 is the only year that is statistically different from the others.

the simple preliminary analysis we use a binary, un-row-standardized[24] contiguity[25] weights matrix.

We first compare the results of the non-spatial OLS model (first column) to the SAR model (second column) in Table 3. The results clearly show that ignoring spatial dependence forces OLS to overestimate the immediate impacts of the covariates; almost all of the OLS coefficients are larger (either more negative or more positive) than their SAR counterparts (Ward and Gleditsch 2008: 68-69). Most notably, the non-spatial OLS would lead scholars to falsely conclude that there is no spatial dependence in states' defense burdens. The SAR model corrects this mistake. As expected, our estimate of the global spatial autocorrelation coefficient, ρ, is statistically significant and positive, indicating that the defense burdens of contiguous states are positively correlated. In other words, we have found evidence that increases to one state's defense burden *simultaneously* induce increases in their neighbors' defense burdens (see also Flores 2011). The result is that covariates, such as civil and interstate wars, influence defense burdens directly (via the βs) and indirectly through spatial diffusion (via ρ).[26]

[Table 3 about here.]

However, as we discussed above, the various assumptions underpinning SAR models may not be consistent with how these processes actually work. Most notably, it is possible that to the extent that spillovers occur, they occur with a delay. Indeed, due to incrementalism in budgeting, defense outlays are likely to be highly path dependent and thus responsive to other countries' outlays with a temporal lag. In other words, if states are responsive

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[24]As Neumayer and Plümper (2016) note, the decision to row-standardize the weights matrix should follow closely from one's theory. In the case of defense burdens, we believe that all countries are not equally influenced by their neighbors' burdens. Instead, the degree to which one state is influenced is based partly on the number of neighbors.

[25]Only those states that are contiguous by land or river are coded as neighbors, according to the Correlates of War project.

[26]In the *Supplemental Materials* we move beyond the coefficients to provide a more in-depth exploration of total, direct and indirect effects.

to the spending patterns by other states, responses will occur one-year later rather than simultaneously. Based on this clear distinction, there is ample justification for estimating an SLX model with temporal lags rather than with an endogenous SAR model where spatial effects are instantaneous.

In the case of defense spending, we believe that the SLX model offers the added benefit of allowing one to properly model spatial heterogeneity. More specifically, as outlined above, the SLX model offers a great deal of flexibility in specifying the particular spatial patterns (if any) that are theorized to govern different variables. This heterogeneity can be accomplished by determining which other countries are important spatially (via the **W**) and by allowing the strength (and sign) of the spatial clustering to vary by variable. This flexibility is particularly helpful when it comes to modeling conditional patterns of spatial dependence, and varying degrees of higher-order effects. We think this is more appropriate than estimating one global spatial autocorrelation coefficient (ρ) that dictates the same spatial process for all the variables.

To demonstrate the flexibility in modeling various spatial patterns, we add the SLX variables derived from three weights matrices (contiguity, alliance commitments and defense pacts) to reflect the following expectations:

- We expect that civil wars in neighboring states will have an impact on states' defense burdens, but there are possibly conflicting reasons for the impact (Phillips 2014). To account for these spatial patterns, we include *contiguity*×*civil war$_{t-1}$*.

- States are likely to increase their defense burdens in response to interstate wars that either increase their own instability or demand that they fulfill alliance commitments. To account for these expectations, we include *contiguity*×*interstate war$_{t-1}$* and *alliance*×*interstate war$_{t-1}$*.

- Temporally-lagged spatial lag (TLSL) variables measure the influence of neighbors' defense burdens in the previous year. We believe that states will respond positively

to prior military spending patterns by both contiguous states and those states with which they share a defense pact. We include *contiguity×defense burden$_{t-1}$* and *defense pact×defense burden$_{t-1}$*.

Results from the SLX model (Model 2 in Table 3) show that a flexible estimation technique (such as SLX) is necessary to derive accurate inferences about spatial patterns. Recall that the SAR model is limited to one specification of the neighbors' connections (via **W**) and one global spatial autocorrelation parameter (ρ).[27] The SLX model shows that both of these constraints are questionable at best. It reveals that countries' defense burdens are connected in more than one way; a single covariate (such as *defense burden$_{t-1}$*) can exhibit spatial dependence through multiple weights matrices (in this case, contiguity and defense pacts), and the nature of this connectivity may differ across covariates (i.e., *interstate war$_{t-1}$* and *defense burden$_{t-1}$*).

The second constraint of the SAR model—that the single ρ parameter accurately reflects the patterns of spatial dependence—is also potentially damaging. The coefficients for the spatial estimates (θs) are statistically different (at the 95% confidence level) in some cases, which means that, for example, the effects of contiguity are more negative for civil wars than interstate wars, and interstate war can influence defense burdens differently based on either contiguity or alliance patterns.[28] Since the coefficients for the two SLX variables for *defense burden$_{t-1}$* are in the opposite direction as the direct effect, it points to a different spillover story than the one from the SAR model. In both models, increases in i's *defense burden* at $t - 1$ lead to decreases at time t for i ($\beta = -0.14$ in the SLX model and $\beta = -0.12$ in the SAR model). In the SLX model, an increase in i's defense burden at $t - 1$ positively spills over in the defense burdens of i's neighbors at t ($\theta_1 = 0.006$ and $\theta_2 = 0.003$); in the SAR

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[27]This is the same obstacle that Flores (2011) faces. He makes a number of statements suggesting that countries' responses to their neighbors' defense burdens are conditioned by circumstances such as alliance commitments and neighborhood size. These patterns of conditional spatial dependence are quite reasonable, but Flores (2011) is unable to make those inferences from SAR models with one global autocorrelation parameter.

[28]In the former case, F-tests suggest that we can reject the null of equal coefficients at the 99% confidence level; in the latter case, we can reject the null of equal coefficients at the 95% confidence level.

model, an increase in i's defense burden at $t-1$ decreases its defense burden at t, which leads to decreases in i's neighbors due to positive spatial dependence ($\rho = 0.07$). The two models therefore give contrasting explanations of the influence of *defense burden* in neighbors depending on whether y_j is influenced by x_i (in the SLX), or y_i (in the SAR). The fact that the direct and indirect coefficients are different signs in the SLX model implies that the SAR model (with its global autocorrelation coefficient) cannot accurately reflect the spatial patterns present in *defense burden*$_{t-1}$.

This exploration of defense spending highlights a significant advantage of SLX models related to spatial heterogeneity. In this case, states' defense burdens are spatially dependent in more than one way, and the strength of spatial dependence varies across connection type and covariate. Both of these—combined with the ability to model higher-order and conditional spatial effects[29]—suggest that SLX models offer the key to unlocking a treasure trove of spatial inferences.

Paths Forward

From our discussion and findings above it is clear that we are advocates of the simplicity offered by the SLX approach. Our goal, however, is not to suggest that the SLX is a superior model for all spatial econometric enterprises. Rather, we advocate for this approach based on several important realities when it comes to the adoption of spatial models in political science. First, many applied researchers are not exposed to spatial models in a way that is complementary to their training (i.e., building from basic linear models). Likewise, many researchers interested in spatial models likely find the current offerings of software tools insurmountable to getting started.[30] Without more accessibility widespread adoption is less

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[29]In the *Supplemental Materials* we first reveal that the extent to which civil wars drive military spending in nearby states depends on the particular region, and then demonstrate how to incorporate higher-order effects in the SLX model.

[30]It is worth pointing out that this is changing with more recent releases of Stata and continual updating of the available R packages. On the other hand, estimation and interpretation are still far less straightforward

likely. How then, should applied researchers proceed when choosing the spatial model that best fits their theory?

A variety of typologies exist for categorizing spatial processes (e.g., Darmofal (2015); Vega and Elhorst (2015)), but a particularly elegant typology is demonstrated by Cook, Hays, and Franzese (2015). The authors note that spatial dependence can arise from spatial clustering in the outcomes (i.e., $\rho\mathbf{W}y$), spatial clustering in the observables (i.e., $\mathbf{WZ\theta}$), or spatial clustering in the unobservables (i.e., $\lambda\mathbf{W}\mu$).These typologies demonstrate, among other things, that there are more than just two models to consider on the spatial econometric menu. Without repeating the full listing, several of these models have particular relevance for our discussion in this paper—especially when it comes to model choice. The so-called Spatial Durbin model (SDM) is in some ways a middle ground between an SLX and SAR because it allows for modeling spatial dependence both between outcomes (e.g., y_j, y_i) and between predictors and outcomes (i.e., x_j, y_i) (Elhorst 2014: 9):

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \rho\mathbf{W}\mathbf{y} + \mathbf{WZ}\boldsymbol{\theta} + \boldsymbol{\epsilon} \tag{14}$$

Cook, Hays, and Franzese (2015) suggest starting with a more general spatial model and then testing the restrictions to pare down the model of unnecessary spatial components. Since it is difficult to recover estimates of the general nesting spatial (GNS) model (i.e., one with estimates of all three spatial parameters) due to weak identification, Cook, Hays, and Franzese (2015) suggest two strategies depending on the goal of the project. If there is concern about accurately characterizing the spatial process, then a spatial autocorrelation model is appropriate; if there is concern about getting accurate estimates of the effects of covariates, then the spatial Durbin model is appropriate. Our approach is similar in that we advocate thinking carefully about theory and whether there are any expectations for higher-order effects, specifying the theoretically-motivated model, and then testing the accuracy

than competing approaches of dealing with spatial heterogeneity—many of which aim to treat it is as nuisance rather than substance.

of those model specifications. If any differences between our approaches arise, it is the point of departure—which we advocate in most political science applications is more closely approximated by the SLX model. This is especially the case in the context of the SAR being the only relatable alternative.

Our advice for the applied researcher is to first follow theory and apply the most analogous spatial model. If for example, the goal is to account for all spatial dependence then the SDM and/or other alternatives (e.g., spatial error models) that capture and eliminate spatial noise in the unobservables may be more useful. It is often the case, however, that one's theory does not provide clear guidance and available diagnostics are uncertain. Which of the two prominent models is less prone to inferential errors as a result of misspecifying the type of dependence? In the *Supplemental Materials* document we provide evidence that the SLX model is more robust to misspecifying the source of the spatial dependence. Additional Monte Carlo analysis based on a DGP with dependence in the error terms (one that is consistent with a spatial error model) shows that simple SLX specifications correctly recover true average effects of variables at expected rates. SAR models, on the other hand, have unsatisfactory recovery rates (i.e., much lower than 95%) for first-order indirect effects, and as a result, the total effects. The SAR is only really appropriate for those theories that specify global dependence in the outcomes among all units—something that we demonstrate is relatively rare in political science. When in doubt, political scientists are well-served to start with an SLX.

Conclusion

The increased use of spatial models in political science is a welcome trend that involves the relaxation of the incredible assumption that there are no contagion or spillover effects across our observations. However, as our review of this literature demonstrates, there is often a substantial disjuncture between expressed theories of spatial relationships and the models

that political scientists estimate. This disjuncture has been particularly prevalent in projects that use the SAR model. We have shown that the SLX model is a good starting point for spatial econometrics in political science because of its flexibility in model specification, ease in estimation, and simplicity in interpretation.

Building on previous works, we provide a comprehensive comparison of the limits of both the SAR and SLX models using simulations. Our Monte Carlo analyses indicate that, even if the true DGP is SAR, the SLX performs quite well at detecting spatial relationships at typically-observed levels. The same cannot be said of the SAR when the true DGP is SLX. This is especially the case if there are even relatively small amounts of heterogeneity in terms of the spatial effects across independent variables in the DGP. Our suggested approach places the power of specification in the researcher's hands and the ability to turn unrealistically imposed assumptions from SAR models into testable propositions.

Our findings in this study lead us to a set of four considerations that researchers should keep in mind when employing spatial models to test their theories. First, researchers should carefully consider the nature of spatial dependence in their theories and choose a model that best reflects their expectations of how these processes work. Second, unless their theories are about global dependence among outcomes, they should start with a simple SLX model and work from there towards more complex models and specifications as appropriate. Third, when they suspect that the spatial diffusion processes are heterogeneous, researchers should employ the SLX with varied specifications of matrices based on these expectations. And fourth, researchers should move beyond a cursory discussion of the direction and statistical significance of the spatial parameters to an interpretation of all the estimated effects (i.e., direct, indirect and total).

Although we have critiqued the choice of the SAR model for many political science applications, we do not begrudge its use under the proper circumstances—theorized endogenous and global dependence among outcomes. In this paper, we aimed to move the spatial rev-

olution in political science to a place where researchers are more easily able to implement, interpret, and understand the applications. The SLX model fills this void by providing a more flexible mapping onto most political science theoretical expectations and by presenting fewer challenges to specification and interpretation.

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Table 1: Applications of Spatial Econometrics in Political Science

	Spatial Theory Type	
Model Used	Global	Local
SAR	80.0%	46.1%
	(4)	(41)
SLX	0.0%	38.2%
	(0)	(34)
Complex	20.0%	10.1%
	(1)	(9)
Other	0.0%	5.6%
	(0)	(5)

Note: This table summarizes the 94 studies citing Beck, Gleditsch and Beardsley (2006) and/or Franzese and Hays (2007) that reported results from at least one spatial econometric model. Cell entries are column percentages on top of raw totals in parentheses.

Table 2: Recovery Rates for Various SLX Model Specifications: SAR Data-Generating Process

	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
Simple SLX model: $\mathbf{y} = \mathbf{x}\beta + \mathbf{Wx}\theta + \epsilon$									
Direct: Total	94.9	94.4	94.8	93.7	95.5	93.5	95.0	94.0	88.9
0 Order	77.9	91.9	95.1	93.5	95.5	93.4	94.8	90.7	81.0
2nd Order	–	–	–	–	–	–	–	–	–
3rd Order	–	–	–	–	–	–	–	–	–
Indirect: Total	15.1	49.9	85.2	93.7	95.2	93.5	40.3	0	0
1st Order	91.4	93.8	95.5	93.1	95.2	94.5	94.4	93.8	93.1
2nd Order	–	–	–	–	–	–	–	–	–
3rd Order	–	–	–	–	–	–	–	–	–
SLX model with a squared term: $\mathbf{y} = \mathbf{x}\beta + \mathbf{Wx}\theta_1 + \mathbf{W^2x}\theta_2 + \epsilon$									
Direct: Total	95.0	94.0	95.1	93.8	95.4	94.0	95.2	94.4	91.0
0 Order	94.2	95.1	95.2	94.2	93.6	94.8	94.8	94.9	95.8
2nd Order	93.2	93.9	94.7	94.6	94.0	94.7	93.6	93.4	93.8
3rd Order	–	–	–	–	–	–	–	–	–
Indirect: Total	93.0	94.7	94.8	94.7	93.9	94.5	93.3	83.7	2.6
1st Order	93.0	94.9	95.9	94.0	94.9	94.9	94.4	93.2	92.6
2nd Order	93.2	93.9	94.7	94.6	94.0	94.7	93.6	93.4	93.8
SLX model with squared and cubed terms: $\mathbf{y} = \mathbf{x}\beta + \mathbf{Wx}\theta_1 + \mathbf{W^2x}\theta_2 + \mathbf{W^3x}\theta_3 + \epsilon$									
Direct: Total	95.0	94.2	94.6	94.3	96.1	94.4	94.5	93.7	93.4
0 Order	94.2	95.1	95.3	94.1	93.9	94.9	94.3	94.9	95.9
2nd Order	93.9	94.0	94.8	95.2	94.2	95.2	93.8	93.2	93.3
3rd Order	94.2	93.6	94.7	93.5	95.0	94.4	95.0	93.9	93.4
Indirect: Total	95.5	95.1	95.6	94.4	95.2	94.5	93.9	92.6	87.0
1st Order	94.3	94.7	95.1	93.9	94.9	93.7	93.9	95.2	94.4
2nd Order	93.9	94.0	94.8	95.2	94.2	95.2	93.8	93.2	93.3
3rd Order	94.2	93.6	94.7	93.5	95.0	94.4	95.0	93.9	93.4

Table 3: Non-Spatial OLS, SAR and SLX Models of Neighborhood Effects on Defense Burdens

	OLS Model 1	SAR Model 2	SLX Model 3
Spatial Estimates (ρ and θ)			
ρ		0.07***	
		(0.004)	
Contiguity×Civil War$_{t-1}$			-0.11*
			(0.07)
Contiguity×Interstate War$_{t-1}$			0.18***
			(0.07)
Ally×Interstate War$_{t-1}$			-0.006
			(0.03)
Contiguity×Defense Burden$_{t-1}$			0.006***
			(0.002)
Defense Pact×Defense Burden$_{t-1}$			0.003***
			(0.001)
Non-Spatial Estimates (β)			
Civil War$_{t-1}$	-0.47***	-0.45***	-0.39***
	(0.17)	(0.16)	(0.15)
Interstate War$_{t-1}$	0.46***	0.45***	0.37**
	(0.16)	(0.16)	(0.16)
Total Population (Logged)$_{t-1}$	0.03	0.03	0.02
	(0.02)	(0.02)	(0.02)
Alliance with US	-0.21**	-0.22**	-0.15*
	(0.10)	(0.09)	(0.08)
ΔUS Defense Burden	-0.04	-0.03	-0.05
	(0.07)	(0.07)	(0.08)
ΔUSSR/Russia Defense Burden	0.006	-0.01	-0.05***
	(0.02)	(0.02)	(0.02)
US Ally×Δ US Defense Burden	0.14	0.11	0.21
	(0.12)	(0.11)	(0.13)
US Ally×Δ USSR Defense Burden	0.005	0.01	0.02
	(0.03)	(0.03)	(0.02)
Annual Trend	-0.007***	-0.006***	-0.004**
	(0.002)	(0.002)	(0.002)
1992	1.13***	0.74***	-1.26***
	(0.28)	(0.27)	(0.21)
Defense Burden$_{t-1}$	-0.12***	-0.12***	-0.14***
	(0.006)	(0.01)	(0.006)
ΔDefense Burden$_{t-1}$	-0.14***	-0.13***	-0.14***
	(0.01)	(0.02)	(0.01)
Constant	0.46**	0.41*	0.22
	(0.22)	(0.21)	(0.17)
N	6,328	6,328	7,266

*Note:* Models include regional fixed effects. The SAR model excludes isolates.
* p-value < 0.1; ** p-value < 0.05; *** p-value < 0.01

Figure 1: Spatial Arrangement with Associated Squared ($\mathbf{W}^2$) and Second-Order ($\mathbf{W}_{2nd}$) Weights Matrices

Map	**W**	**W²**	**W₂ⁿᵈ**

Map

W

	A	B	C	D	E	F
A	0	1	1	1	0	0
B	1	0	0	1	1	0
C	1	0	0	1	0	0
D	1	1	1	0	0	1
E	0	1	0	0	0	0
F	0	0	0	1	0	0

First-Order Neighbors

W²

	A	B	C	D	E	F
A	3	1	1	2	1	1
B	1	3	2	1	0	1
C	1	2	2	1	0	1
D	2	2	1	4	1	0
E	1	0	0	1	1	0
F	1	1	1	0	0	1

First-Order Neighbors (Squared)

W₂ⁿᵈ

	A	B	C	D	E	F
A	0	0	0	0	1	1
B	0	0	1	0	0	1
C	0	1	0	0	0	1
D	0	0	0	0	1	0
E	1	0	0	1	0	0
F	1	1	1	0	0	0

**Second-Order Neighbors**

Figure 2: Recovery of the True Total Effects of x_1 in the SAR Model across Strength of Spatial Autocorrelation and Spatial Heterogeneity
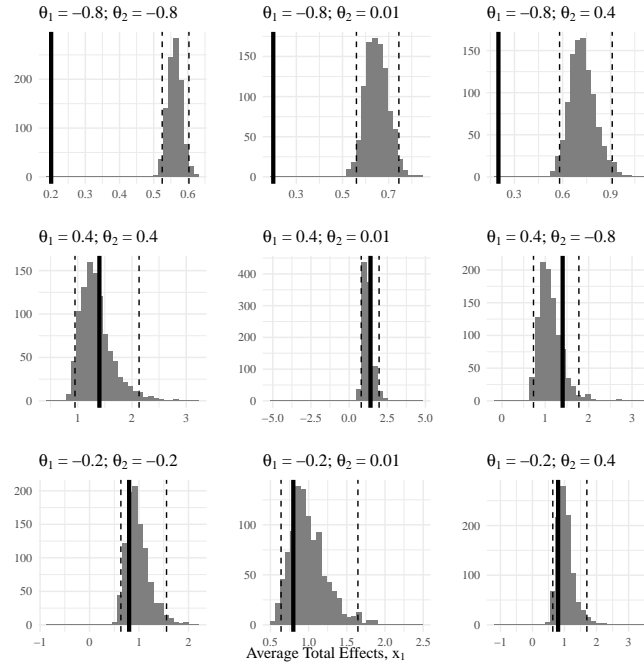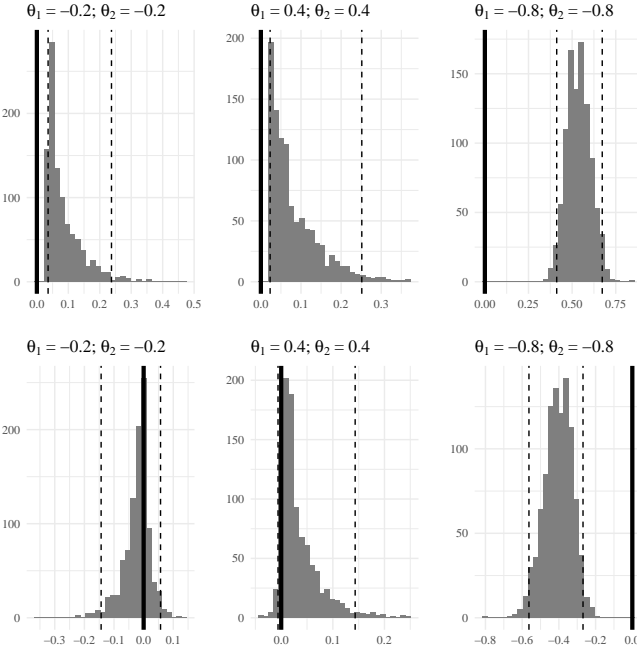

Note: θ values refer to those from the SLX Monte Carlo simulations. Vertical lines represent 95% confidence intervals (dashed) and the true total effect of x_1 (solid).

Figure 3: Recovery of the True Indirect Effects of x_2 in the SAR Model across Strength of Spatial Autocorrelation and Spatial Heterogeneity
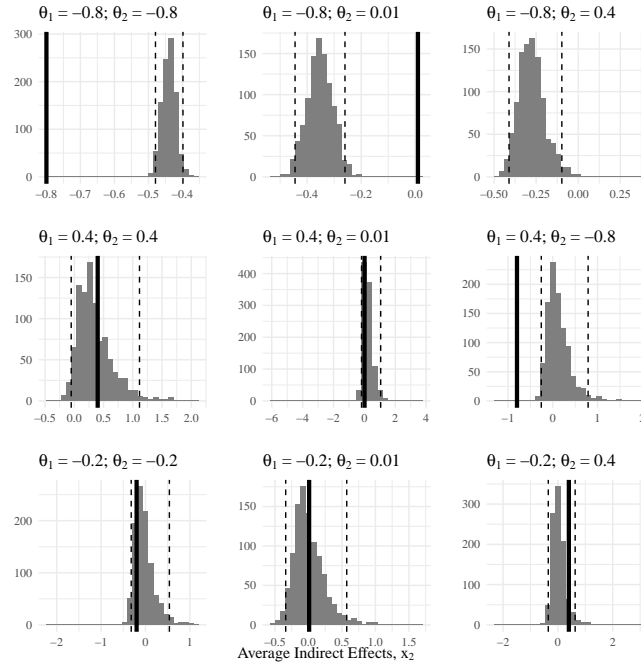
Note: θ values refer to those from the SLX Monte Carlo simulations. Vertical lines represent 95% confidence intervals (dashed) and the true indirect effect for x_2 (solid).

Figure 4: SAR Model Indicates the Presence of Higher-Order Indirect Effects for x_1 When They Are Not There

Note: θ values refer to those from the SLX Monte Carlo simulations. Vertical lines represent 95% confidence intervals (dashed) and the true higher-order effects for x_1 (solid).